

## Alternative methods for determining the altitude of theodolite observation stations

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Theodolites have been used for many years to measure the movement patterns of near-shore groups of marine mammals (Würsig *et al.* 1991). This land-based method allows groups of animals to be tracked without affecting their behavior, which is important when looking at effects of potential disturbance on marine mammals (*e.g.*, Bejder *et al.* 1999, Cox *et al.* 2003) or documenting their normal behavior (*e.g.*, Bailey and Thompson 2006). Theodolite tracking also provides a method for measuring the distribution and relative abundance of animals over time (*e.g.*, Harzen 2002, Gailey *et al.* 2007).

The basic method for determining the location of a target (*e.g.*, whale, dolphin, or vessel) is to measure the angles of declination and bearing from the shore station to the waterline of the target. With a known elevation above sea level, the declination angle can be converted to a distance (Lerczak and Hobbs 1998). Thus distance measurement accuracy is a direct function of elevation measurement accuracy. The distance, combined with the bearing, provides the polar coordinates from the shore station to the target object. The polar coordinates can then be converted to Cartesian coordinates relative to the shore station or into latitude and longitude values. Compass-equipped reticle binoculars can provide acceptable measures of location in some situations (Yin *et al.*, unpublished data), but more typically the measured angles have been obtained with a theodolite (Mendes *et al.* 2002, Bejder *et al.* 1999).

In this note we review methods used to determine total shore station elevation, including a previously unpublished method, and introduce a new method for determining height. For convenience, total shore station elevation is typically divided into three components:

$$\text{Total Shore Station Elevation} = \text{Shore Station Height (above mean low water)} \\ + \text{Eye Height} + \text{Tide Height}$$

At low-lying shore stations, the total elevation can be determined by comparison to a previously surveyed marker (*e.g.*, Best *et al.* 1995) or directly with the geometric leveling technique.<sup>1</sup> In this approach, a stadia rod (*i.e.*, a marked and measured pole) is placed at the waterline. The theodolite telescope is set to a 90° declination, and the station elevation can be read directly from the stadia rod. However, many observation stations are too high to allow a single direct measurement. A method to overcome this problem is called “leapfrogging.” The total elevation is divided into a series of partial elevation measurements from the water level to the shore station. While this method is straightforward, it has a greater potential for error due to the number of individual measurements and repeated placements of the stadia rod. Würsig *et al.* (1991) presented a method for measuring shore height that relies on visual measurements of points a known distance apart on the shoreline, and this method was extended by Bailey and Lusseau (2004).

Another trigonometric method was implemented for the University of Hawai‘i (UH) humpback whale project in the early 1980s. This previously unpublished method also requires the ability to access the waterline and the ability to view the entire stadia rod from the shore station. The stadia rod is held vertically at the water’s edge and the theodolite operator then measures the declination angle from the theodolite to the top and bottom of the pole. At least 10 measurements, alternating top and bottom, are recommended to reduce measurement error, with the mean taken of all measurements. These angles are then used to determine the total elevation of the theodolite.

The theodolite operator also measures the eye height, or height of the pivot point of the theodolite telescope above the ground. Lastly, the tide height needs to be measured or predicted using tide stakes and/or tide tables and added or subtracted from the total elevation.

The algorithm to calculate the total elevation is presented below and illustrated in Figure 1. Note that this formulation is based upon a frame of reference where 0° is straight up, 90° is horizontal, and the angles to the target are greater than 90°.

Angle B = mean measured bottom angle

Angle T = mean measured top angle

Angle A = Angle B – Angle T

Hypotenuse = Pole Height × sine (Angle T)/sine (Angle A)

Station Height = Hypotenuse × sine (Angle B).

A program to perform the UH calculation is posted at [www.hmmc.org/theoHeight.html](http://www.hmmc.org/theoHeight.html).

This approach uses the law of sines in order to find the hypotenuse of the right triangle. Total shore station elevation is then the product of this hypotenuse value

<sup>1</sup>Krogman, B. D., and D. J. Rugh. 1983. Instructions for conducting a census of bowhead whales from ice-based observation sites near Point Barrow, Alaska. Available from the National Marine Mammal Laboratory, 7600 Sand Point Way N.E. F/AKC3, Seattle, WA 98115-6349.

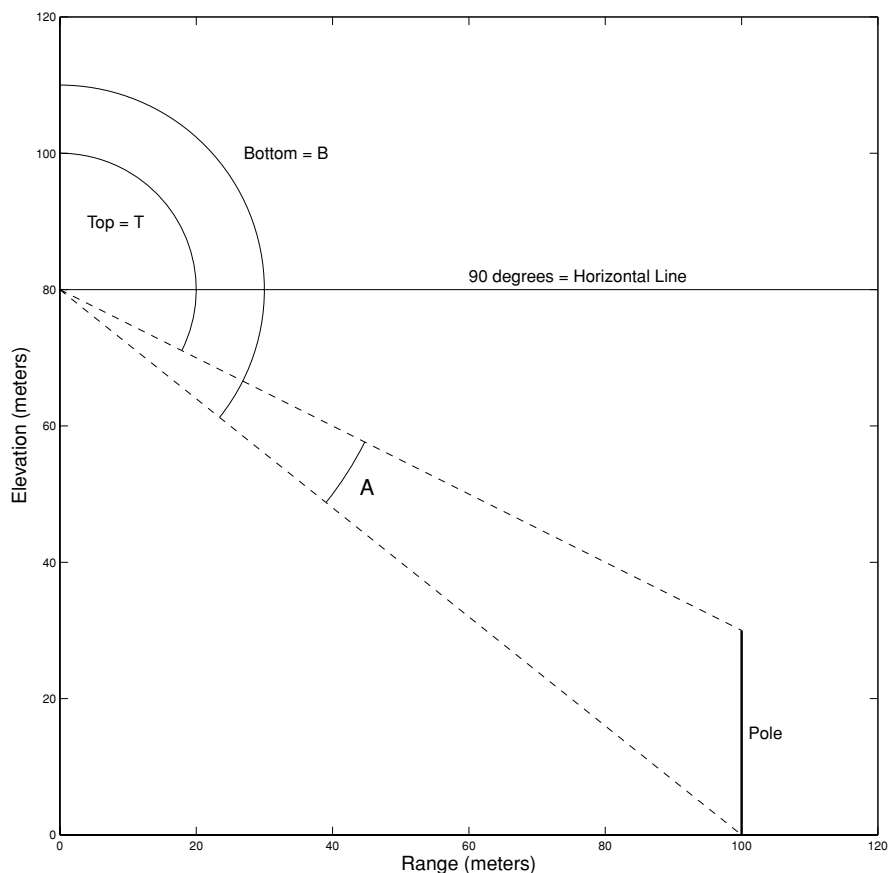


Figure 1. The UH method for determining elevation. The values on the axes are hypothetical. The three measurements needed to calculate elevation are the declination angles to the top and bottom of the pole, and the pole height. Angle A and the total elevation are calculated from these three measurements.

multiplied by the sine of the bottom angle. The hypotenuse can also be measured directly with the use of a total station. A total station incorporates an electronic distance meter with a theodolite. While it is simpler to use a total station, they may not be available to the researcher.

The UH algorithm as well as the Würsig *et al.* (1991) approach have an intrinsic error in that they do not account for the curvature of the earth. Typically, the small distance from the theodolite to the stadia rod produces an insignificant error. However, the new approach presented here does account for the curvature of the earth, and may be the preferred method for determining the height of shore station set significantly back from the shoreline.

During the analysis of data from a calibration experiment comparing locations determined by Global Positioning System (GPS) and theodolite (*e.g.*, Mendes *et al.* 2002), that was expanded to include reticle binocular distance estimation (Yin *et al.*,

unpublished data), we realized that the combination of simultaneous GPS-derived vessel positions and theodolite fixes on that vessel could be used to measure the total height of eye. A new algorithm was developed for this purpose and is presented here as a complement to the previously published Würsig *et al.* (1991) method.

First, the latitude and longitude of the shore station are determined. In our study we collected GPS measurements with a GPS base station, sampling every second, collecting a total of 9,513 position measurements. We then calculated the geometric mean of latitude and longitude that serves as a reference point.

Next, we positioned our research vessel in front of the shore station. Personnel aboard the research vessel recorded the GPS-determined vessel position while the theodolite operator simultaneously took a fix at the vessel's waterline. A single GPS position was considered sufficient since we were using a Wide Area Augmentation System (WAAS) enabled GPS receiver with an average location error of  $<3$  m. Radio communications between shore and vessel facilitated coordinated data collection. The shore station crew then directed the research vessel to a wide range of bearings and distances from the shore station. This provided multiple simultaneous paired position measurements to reduce measurement error and account for any effect of bearing or range. In order to minimize measurement error, we elected to collect data on a day with little or no swell and good visibility (*i.e.*, clear horizon with little glare or haze).

Once the data had been collected, the distance between the shore station and each GPS boat location was calculated using a Matlab function (`dist.m`) that accounts for a spherical earth (Newhall 1997). These were compared to distances calculated from the theodolite data. The conversion from declination angle to distance requires an (unknown) elevation value. Therefore a range of candidate elevation values was used to calculate distance measurements. Elevation values that were too low underestimated the distance to the vessel, while elevations that were too high produced overestimates. Because the magnitude of the distance error is a function of target distance (see Fig. 2), the differences in distance estimates were normalized by the measured GPS distance. The error term is defined as the difference between the GPS-based and theodolite data-derived distance measures, divided by the GPS distance. The elevation value that produced the lowest root-mean square (rms) error was considered the true total shore station elevation.

This procedure was repeated with an increased precision of the candidate elevation values. The first iteration employed a wide range of candidate shore station elevations with a resolution of 1 m. This first rough measurement was conducted to determine the approximate value of station height. The procedure was then repeated, with 200 candidate elevation values ranging from 1 m above to 1 m below the approximate value with a resolution of 0.01 m. This returned the value for the total shore station elevation, and the shore station height can be extracted by subtracting out the tide height and the eyepiece height.

To illustrate this procedure, the results of our calibration experiment are provided here. On 13 March 2004 we took theodolite fixes on our research vessel at 43 locations. A simultaneous GPS-measured location was obtained for each location. A Matlab program was written to conduct the analysis described above. Figure 2

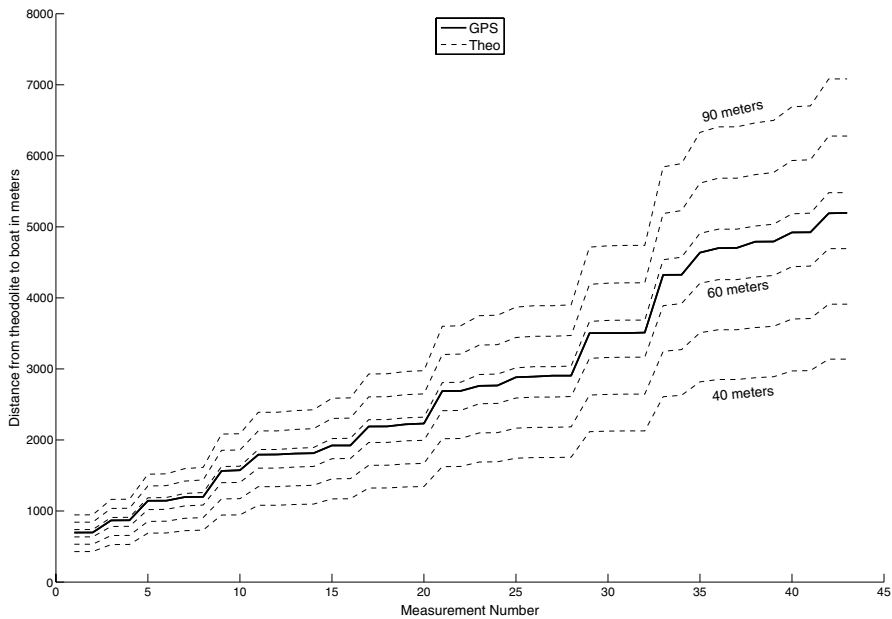
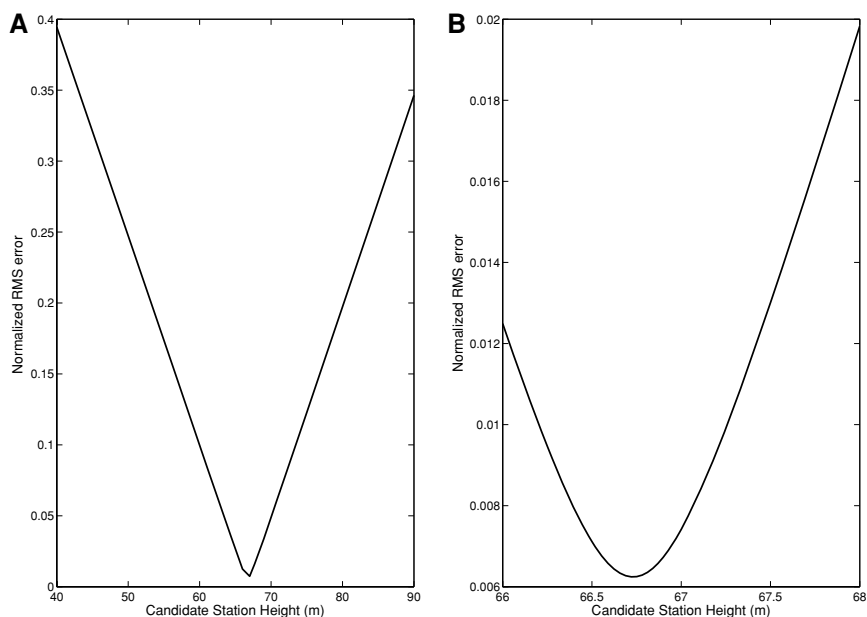


Figure 2. A series of 43 different GPS-measured vessel locations and theodolite angle measurements were taken. The thick line shows the GPS-measured distances. The series of thin lines represent the distances calculated using candidate shore station elevations ranging from 40 to 90 m. It can be seen that the difference between measured and calculated distance is proportional to distance.

illustrates the results of this first analysis using a range of candidate station heights from 40 to 90 meters. The error term (*i.e.*, the difference between the GPS-derived distance and theodolite-derived distance) was examined, and the minimum error occurred with a candidate shore station altitude of 67 m (see Fig. 3A). The program was then rerun with candidate height values ranging from 66 to 68 m in steps of 0.01 m. The resulting best estimate for height of eye was 66.73 m (see Fig. 3B). From this the eyepiece height of 1.52 m and a tide height of 0.15 m were subtracted to give a station height of 65.06 m. This value matches well with our previous calculated estimate of 65.5 m using the UH method. Note that in the UH method measurement, tide height was not considered. The tide height for that morning ranged between 0.2 and 0.4 m (<http://tidesandcurrents.noaa.gov>). This may account for some of the difference between the results of the two methods.

To evaluate the precision of this estimate, the differences between predicted distance and the GPS-based distance were normalized by the GPS distance. The rms of this normalized error term was 0.0067. Furthermore, a regression of calculated distance to measured distance had an  $r^2$  of 0.999. The low error term and high regression  $r^2$  value indicate that this technique is producing an accurate shore elevation estimate.



*Figure 3.* A series of 43 different GPS-measured vessel locations and theodolite angle measurements were taken. Plot A shows the normalized RMS error between the GPS-measured distance and the calculated distance for a series of candidate shore station heights between 40 and 90 m. The minimum error was found at 67 m. Plot B shows the same process repeated, between 66 and 68 m in 0.01-m intervals. The minimum error was measured at 66.73 m.

All of the methods presented and discussed here and in the referenced papers can produce an accurate measurement of shore station height. However, the GPS-comparison method has some advantages over the other approaches. First, it can be used for study sites where the shoreline is inaccessible or not visible to the theodolite operator. Secondly, the GPS-comparison method inherently accounts for curvature of the earth, which may be important for those study sites set well back from the shoreline. The GPS-comparison method also avoids the repeated measures and potential for error inherent in the leapfrog approach. In our test we deployed the GPS on a vessel as part of another experiment, which requires the vessel and radio communications to synchronize theodolite and GPS data. However, the GPS method can be used without a vessel at sites where the shoreline is both visible and accessible. The GPS antenna can be placed at the waterline, and it can be fixed by the theodolite operator, producing the same type of data as the boat-based approach. We conclude that a simultaneous GPS method with theodolite measurements is a useful addition to other published methods of determining shore station height, especially in areas where the shoreline is not easily accessible or visible.

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